

Kaon condensation in CFL quark matter, the Goldstone theorem, and the 2PI Hartree approximation

Lars E. Leganger

Department of Physics, Norwegian University of Science and Technology, Høgskoleringen 5, N-7491 Trondheim, Norway

Abstract. At very high densities, QCD is in the color-flavor-locked phase, which is a color-superconducting phase. The diquark condensates break chiral symmetry in the same way as it is broken in vacuum QCD and gives rise to an octet of pseudo-Goldstone bosons and a superfluid mode. The lightest of these are the charged and neutral kaons. For energies below the superconducting gap, the kaons are described by an $O(2) \times O(2)$ -symmetric effective scalar field theory with chemical potentials. We use this effective theory to study Bose-condensation of kaons and their properties as functions of the temperature and the chemical potentials. We use the 2-particle irreducible effective action formalism in the Hartree approximation. The renormalization of the gap equations and the effective potential is studied in detail and we show that the counterterms are independent of temperature and chemical potentials. We determine the phase diagram and the medium-dependent quasiparticle masses. It is shown that the Goldstone theorem is satisfied to a very good approximation.

Keywords: kaon condensation, color-flavor-locked phase, quark matter, goldstone theorem, 2PI Hartree approximation

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INTRODUCTION

At high density and low temperature, we know that QCD is in the color-flavor-locked (CFL) phase [1, 2, 3, 4]. This state is a color superconducting state since the quarks form Cooper pairs as electrons in an ordinary superconductor. The attraction between the quarks, which renders the Fermi surface unstable against the formation of Cooper pairs, is provided by one-gluon exchange.

At asymptotically high densities, one can ignore the strange-quark mass and quarks of all three colors and all three flavors participate in a symmetric manner in the pairing. The original symmetry group $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$ is broken down to $SU(3)_{c+L+R}$ which is a linear combination of the generators of the original group. This locks rotation in color space with rotations in flavor space and this has given the name to the phase. Since the symmetry-breaking pattern is the same as in vacuum QCD, the low-energy properties of the CFL phase can be described in terms of an effective chiral Lagrangian for the octet of (pseudo-) Goldstone modes and the superfluid mode [5, 6, 7, 8, 9]. An important difference between chiral perturbation theory in the vacuum and in the CFL phase is that the latter is at high density and the Lagrangian is therefore coupled to chemical potentials via the zeroth component of a "gauge field".

At asymptotically high densities, all modes are exactly massless since one can ignore the quark masses. At moderate densities, this is no longer the case. The quark masses cannot be neglected and chiral symmetry is explicitly broken. This implies that only the super-

fluid mode is exactly massless, while the other mesonic modes acquire masses. This is relevant for the interior of a neutron star. In this case, the quark chemical potential is of the order of 500 MeV, while the strange quark mass is somewhere between the current quark mass of approximately 100 MeV and the constituent quark mass of approximately 500 MeV [10]. The mass spectrum in the CFL is the opposite of vacuum QCD and the lightest massive modes are expected to be the charged and neutral kaons K^+/K^- and K^0/\bar{K}^0 .

The 2PI effective action formalism [11] has recently been used to study the thermodynamics of pions and kaons and their condensation. In Ref. [10], the authors applied the 2PI effective action formalism in the Hartree approximation to an effective $O(2) \times O(2)$ -symmetric scalar field and calculated the phase diagram and the critical temperature for Bose-condensation of kaons. The effects of imposing electric charge neutrality were also investigated. The scalar theory for the kaons were derived from the effective chiral Lagrangian, where the parameters depend on the baryon chemical potential. Renormalization issues were not addressed.

In the present paper we reconsider the problem of kaon condensation from a somewhat different angle, and consider the renormalization of the theory. In particular, we find that all divergences in the gap equations and the effective potential can be eliminated by counterterms that are independent of temperature and chemical potentials. We also show that the violation of Goldstone's theorem is negligible. A more detailed presentation of the calculations can be found in [12].

KAONS IN THE CFL PHASE

The chiral effective Lagrangian of dense QCD in the CFL phase is given by [6]

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} f_\pi^2 \text{Tr} [(\partial_0 \Sigma + i[A, \Sigma]) (\partial_0 \Sigma - i[A, \Sigma]^\dagger) \\ & - v_\pi^2 (\partial_i \Sigma) (\partial_i \Sigma^\dagger)] \\ & + \frac{1}{2} a f_\pi^2 \det M \text{Tr} [M^{-1} (\Sigma + \Sigma^\dagger)] + \dots, \quad (1) \end{aligned}$$

where f_π , v_π , and a are constants, the meson field $\Sigma = e^{i\lambda^a \phi^a / f_\pi}$, where λ^a are the Gell-Mann matrices and ϕ^a describe the octet of Goldstone bosons. The matrix $A = \mu_Q Q - \frac{M^2}{2\mu}$ acts as the zeroth component of a gauge field, where μ_Q is the chemical potential for electric charge Q , μ is the baryon chemical potential, $Q = \text{diag}(2/3, -1/3, -1/3)$, and $M = \text{diag}(m_u, m_d, m_s)$. At asymptotically high densities, one can use perturbative QCD calculations to determine the parameters by matching [6, 8, 9]. For moderate densities, which are relevant for compact stars, a precise determination of the parameters is difficult.

Expanding to fourth order in the meson fields, one obtains an effective Lagrangian for the kaons where the parameters depend on the quark masses, chemical potentials, and f_π , as determined by matching [10]. Generally these parameters must satisfy some RG equations, but we do not know how they evolve down to values of the chemical potential relevant for a star. We therefore chose to use our effective theory in a more traditional way, where we are content with using coupling constants inspired by the matching.

The kaons are written as a complex doublet, $(K^0, K^+) = (\Phi_1, \Phi_2)$. The Euclidean Lagrangian with an $O(2) \times O(2)$ symmetry is given by

$$\begin{aligned} \mathcal{L} = & [(\partial_0 + \mu_0) \Phi_1^\dagger] [(\partial_0 - \mu_0) \Phi_1] + (\partial_i \Phi_1^\dagger) (\partial_i \Phi_1) \\ & + [(\partial_0 + \mu_+) \Phi_2^\dagger] [(\partial_0 - \mu_+) \Phi_2] \\ & + (\partial_i \Phi_2^\dagger) (\partial_i \Phi_2) + m_0^2 \Phi_1^\dagger \Phi_1 + m_+^2 \Phi_2^\dagger \Phi_2 \\ & + \frac{\lambda_0}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_+}{2} (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_H (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2). \quad (2) \end{aligned}$$

The chemical potentials μ_0 and μ_+ are associated with the two conserved charges for each complex field Φ_i , related to the quark chemical potentials by $\mu_0 = \mu_d - \mu_s$, $\mu_+ = \mu_u - \mu_s$.

In order to allow for a condensate of neutral kaons, we introduce an expectation value ϕ_0 for Φ_1 and write

$$\Phi_1 = \frac{1}{\sqrt{2}} (\phi_0 + \phi_1 + i\phi_2), \quad (3)$$

where ϕ_1 and ϕ_2 are quantum fluctuating fields. The inverse tree-level propagator can be written as a block-diagonal 4×4 matrix $D_0^{-1} = \text{diag}(D_a, D_b)$:

$$D_a = \begin{pmatrix} P_n^2 + m_1^2 - \mu_0^2 & -2\mu_0 \omega_n \\ 2\mu_0 \omega_n & P_n^2 + m_2^2 - \mu_0^2 \end{pmatrix}, \quad (4)$$

$$D_b = \begin{pmatrix} P_n^2 + m_3^2 - \mu_+^2 & -2\mu_+ \omega_n \\ 2\mu_+ \omega_n & P_n^2 + m_3^2 - \mu_+^2 \end{pmatrix}, \quad (5)$$

where $P_n^2 = \omega_n^2 + p^2$ and the tree-level masses are $m_1^2 = m_0^2 + \frac{3\lambda_0}{2} \phi_0^2$, $m_2^2 = m_0^2 + \frac{\lambda_0}{2} \phi_0^2$, and $m_3^2 = m_+^2 + \frac{\lambda_H}{2} \phi_0^2$.

The 2PI effective action is given by

$$\begin{aligned} \Omega[\phi_0, D] = & \frac{1}{2} (m_0^2 - \mu_0^2) \phi_0^2 + \frac{\lambda_0}{8} \phi_0^4 + \frac{1}{2} \text{Tr} \ln D^{-1} \\ & + \frac{1}{2} \text{Tr} D_0^{-1} D + \Phi[D], \quad (6) \end{aligned}$$

where $\Phi[D]$ contains all 2PI vacuum diagrams. In the Hartree approximation, we include all double-bubble diagrams which can be written in terms of $O(2) \times O(2)$ invariants [13]

$$\begin{aligned} \Phi[D] = & \frac{\lambda_0}{8} [\text{Tr}(D_a)]^2 + 2\text{Tr}(D_a^2) + \frac{\lambda_+}{8} [\text{Tr}(D_b)]^2 \\ & + 2\text{Tr}(D_b^2) + \frac{\lambda_H}{4} (\text{Tr} D_a)(\text{Tr} D_b). \quad (7) \end{aligned}$$

The contributions from the sum-integrals can be split into divergent medium-independent and convergent medium-dependent parts, allowing medium-independent counterterms. After renormalization, the self-consistent set gap equations read

$$\Delta M_1^2 = \frac{1}{2} [3\lambda_0 J_1^{c,T} + \lambda_0 J_2^{c,T} + 2\lambda_H J_3^{c,T}], \quad (8)$$

$$\Delta M_2^2 = \frac{1}{2} [\lambda_0 J_1^{c,T} + 3\lambda_0 J_2^{c,T} + 2\lambda_H J_3^{c,T}], \quad (9)$$

$$\Delta M_3^2 = \frac{1}{2} [\lambda_H J_1^{c,T} + \lambda_H J_2^{c,T} + 4\lambda_+ J_3^{c,T}], \quad (10)$$

$$\begin{aligned} 0 = & \phi_0 \left[m_0^2 - \mu_0^2 + \frac{\lambda_0}{2} \phi_0^2 + \frac{1}{2} [3\lambda_0 J_1^{c,T} \right. \\ & \left. + \lambda_0 J_2^{c,T} + 2\lambda_H J_3^{c,T}] \right], \quad (11) \end{aligned}$$

where $\Delta M_i^2 = M_i^2 - m_i^2$ and J 's are medium-dependent convergent integrals involving the dressed masses M .

The renormalized effective potential is given by

$$\begin{aligned} \Omega = & \frac{1}{2} (m_0^2 - \mu_0^2) \phi_0^2 + \frac{\lambda_0}{8} \phi_0^4 + \frac{1}{2} \mathcal{J}_1^{c,T} + \frac{1}{2} \mathcal{J}_2^{c,T} \\ & + \mathcal{J}_3^{c,T} - \frac{1}{2} \Delta M_1^2 J_1^{c,T} - \frac{1}{2} \Delta M_2^2 J_2^{c,T} - \Delta M_3^2 J_3^{c,T} \\ & + \frac{3\lambda_0}{8} (J_1^{c,T})^2 + \frac{3\lambda_0}{8} (J_2^{c,T})^2 + \lambda_+ (J_3^{c,T})^2 \\ & + \frac{\lambda_0}{4} J_1^{c,T} J_2^{c,T} + \frac{\lambda_H}{2} J_1^{c,T} J_2^{c,T} + \frac{\lambda_H}{2} J_2^{c,T} J_3^{c,T}. \quad (12) \end{aligned}$$

RESULTS

In Fig. 1, we show the neutral kaon condensate as a function of μ_0 and μ_+ for $T = 0$. For $\mu_+ = 0$, i.e. along the μ_0 -axis, there is a second-order phase transition to a neutral phase with a kaon condensate at a critical chemical potential $\mu_0 = m_2$. This is the CFL- K^0 phase. For larger values of μ_+ the transition becomes first order, to a phase with condensate of charged kaons. This CFL- K^+ phase condensate is not shown in the figure. Thus there is a competition between the neutral and the charged condensates and nowhere do they exist simultaneously. The transitions to the kaon-condensed phases are density driven. In [12] we also examine effects of imposing electric charge neutrality.

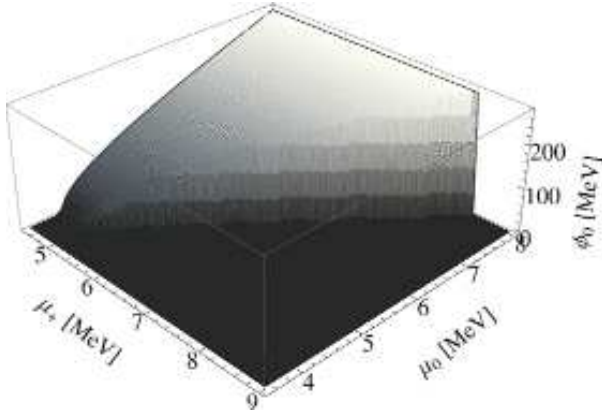


FIGURE 1. Neutral kaon condensate as a function of the chemical potentials μ_0 and μ_+ for $T = 0$.

In Fig. 2 we show the masses of K^0 ($\omega_2(q=0)$) and K^+ ($\omega_4(q=0)$) for $\mu_0 = \mu_+ = 4.5$ MeV and as functions of T normalized to $T_c \sim 120$ MeV. We notice that the mass of K^0 is not strictly zero, which explicitly shows that the Goldstone theorem is not respected by the Hartree approximation. In Ref. [10], the authors make some further approximations of the sum-integrals appearing in the gap equations. These approximations give rise to an exactly gapless mode. As pointed out in their paper and as can be seen in Fig. 2, this is a very good approximation.

SUMMARY AND OUTLOOK

We have studied the phase diagram and the quasiparticle masses of the $O(2) \times O(2)$ model where the neutral kaons condense at sufficiently low temperature and sufficiently large value of the chemical potential, and showed it is possible to renormalize the gap equations and effective potential in a medium-independent way. If the transition is first order, it turns out that the transition is not to a symmetric state but to a state with a K^+ condensate.

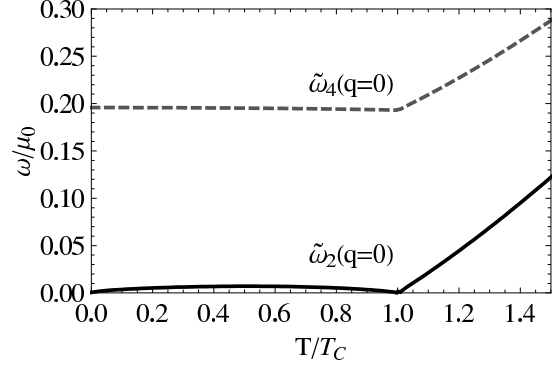


FIGURE 2. Mass gaps for the K^+ and K^0 modes for $\mu_0 = \mu_+ = 4.5$ MeV and as a function of T normalized to T_c .

This is in agreement with the findings of Alford, Braby, and Schmitt [10].

One drawback of the 2PI Hartree approximation is that it does not obey Goldstone's theorem. For practical purposes we find the violation is negligible which is reassuring. We therefore believe that the 2PI Hartree approximation is a useful nonperturbative approximation for systems in thermal equilibrium.

The Hartree approximation and the large- N limit are both mean-field approximations. It would be desirable to go beyond mean field for example by including next-to-leading corrections in the $1/N$ -expansion.

REFERENCES

1. M. G. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).
2. F. Wilczek, "QCD in extreme conditions," arXiv:hep-ph/0003183.
3. K. Rajagopal and F. Wilczek, "The condensed matter physics of QCD," arXiv:hep-ph/0011333.
4. M. G. Alford, A. Schmitt, K. Rajagopal and T. Schafer, "Color superconductivity in dense quark matter," Rev. Mod. Phys. **80**, 1455 (2008).
5. R. Casalbuoni and R. Gatto, Phys. Lett. **B464** 111.
6. D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2001); ibid D **62**, 059902 (2002).
7. P. F. Bedaque and T. Schäfer, Nucl. Phys. A **697**, 802 (2002).
8. D. B. Kaplan and S. Reddy, Phys. Rev. D **65**, 054042(2002).
9. T. Schäfer, Phys. Rev. D **65**, 094033 (2002).
10. M. G. Alford, M. Braby, and A. Schmitt, J. Phys. G: nucl. Part. Phys. **35**, 025002 (2008).
11. J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. **D10**, 2428 (1974).
12. J. O. Andersen and L. E. Leganger, Nucl. Phys. A **828**, 360 (2009) [arXiv:0810.5510 [hep-ph]].
13. G. Fejos, A. Patkos, and Zs. Szep, Nucl. Phys. A **803**, 115 (2008).